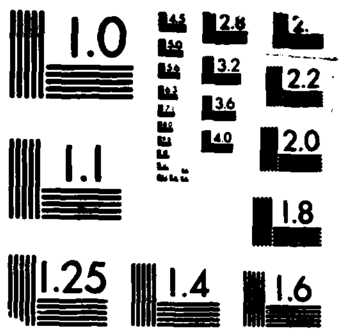
AD-A193 642 LINEARIZED MODEL MATCHING FOR SINGLE-IMPUT NONLINEAR SYSTEMS(U) JOHNS HOPKINS UNIV BALTIMORE MD DEPT OF ELECTRICAL ENGINEERIN.. W J RUGH 01 MAR 88 1/1

UNCLASSIFIED AFOSR-TR-88-0437 AFOSR-87-0101 F/G 12/6 NL



MICROCOPY RESOLUTION TEST CHART
UREAU OF STANDARDS-1963-A

ed
CATION OF THIS PAGE

DTIC FILE COPY (2)

AD-A193 642

DTIC
ELECTE
MAY 0 3 1988
S D
CD

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for Public Release; Distribution unlimited
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 88-0437
NAME OF PERFORMING ORGANIZATION Johns Hopkins University — 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION AFOSR/NM
ADDRESS (City, State, and ZIP Code) Baltimore, Maryland 21218	7b. ADDRESS (City, State, and ZIP Code) Bldg. 410 Bolling AFB, DC 20332-6448
NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR — 8b. OFFICE SYMBOL (If applicable) NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-87-0101
ADDRESS (City, State, and ZIP Code) Bldg. 410 Bolling AFB, DC 20332-6448	10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.
61102F	2304	A1	

11. TITLE (Include Security Classification)

Linearized Model Matching for Single-Input Nonlinear Systems (U)

12. PERSONAL AUTHOR(S)
Rugh, Wilson J.

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Journal Reprint	FROM 3/1/87 TO 2/29/88	1988, March 1	45

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Nonlinear Control Systems, Control Theory

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

A linearized, input-output model matching problem for single-input, multi-output nonlinear systems is formulated in the frequency domain, and solved using dynamic output feedback. The objective is to obtain a desired transfer function for the family of closed-loop linearized systems.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION Unclassified
22a. NAME OF RESPONSIBLE INDIVIDUAL Maj. James M. Crowley	22b. TELEPHONE (Include Area Code) (202) 767-5025 — 22c. OFFICE SYMBOL NM

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

Linearized Model Matching
for Single-Input Nonlinear Systems

Jianliang Wang and Wilson J. Rugh

Department of Electrical and Computer Engineering
The Johns Hopkins University
Baltimore, Maryland 21218
(301)338-7004

Technical Report JHU/ECE 87-14

Linearized Model Matching for Single-Input Nonlinear Systems

Jianliang Wang and Wilson J. Rugh

Department of Electrical and Computer Engineering
The Johns Hopkins University
Baltimore, Maryland 21218
(301)338-7004

Technical Report JHU/ECE 87-14

ABSTRACT

A linearized, input-output model matching problem for single-input, multi-output nonlinear systems is formulated in the frequency domain, and solved using dynamic output feedback. The objective is to obtain a desired transfer function for the family of closed-loop linearized systems.

Introduction

In recent years there has been rapid development of the closely-related pseudo-linearization [3,4] and extended linearization [1,5] approachs for design of nonlinear control laws for nonlinear systems. In this paper we adopt a frequency-domain viewpoint, and use results in [5] to formulate and solve a *linearized model matching* problem for single-input, multi-output nonlinear systems. The objective is to construct a nonlinear, dynamic, output feedback control law so that the resulting closed-loop system, when linearized about its family of constant operating points, has a linearization family with transfer function that matches

Research sponsored by the Air Force Office of Scientific Research, Air Force Systems Command, USAF, under Grant Number AFOSR-87-0101.

exactly a given parameterized transfer function. If the given transfer function is parameter independent, then this problem can be viewed as a type of input-output pseudo-linearization problem.

We consider a single-input, multi-output nonlinear system with input $u(t) \in R$ and output $y(t) \in R^p$, defined for $t \geq 0$. The system is assumed to have a constant operating point family $\{[\mathbf{u}(\alpha), \mathbf{y}(\alpha)],\ \alpha \in \Gamma\}$ where the parameter set $\Gamma \subset R$ is an open neighborhood of a fixed $\alpha_0 \in R$. That is, for a constant input $u(t) = \mathbf{u}(\alpha_1)$, where $\alpha_1 \in \Gamma$ is fixed, there exists an initial condition for the nonlinear system such that the output is the constant, $y(t) = \mathbf{y}(\alpha_1)$. Typically, the constant operating point family is parameterized by constant values of the input, one of the outputs, or one of the underlying states of the system.

We will adopt a local viewpoint throughout this paper and implicitly allow shrinking the size of Γ in order to economically state results. (When particular examples are considered, it often is possible to solve the problem in a nonlocal fashion.) To avoid counting the order of continuous differentiability, we will assume that all functions and their derivatives that appear in the sequel are continuous.

Corresponding to the operating point family $\{[\mathbf{u}(\alpha), \mathbf{y}(\alpha)],\ \alpha \in \Gamma\}$, let the $p \times 1$ parameterized transfer function of the linearization family for the given nonlinear system be

$$T_\alpha(s) = \frac{N(s, \alpha)}{d(s, \alpha)} \tag{1}$$

where $d(s, \alpha)$ and each entry of $N(s, \alpha)$ are polynomials in s with coefficients that are real functions of α, and $d(s, \alpha)$ is monic. We assume that for each $\alpha \in \Gamma$ (1) is irreducible. Also we assume $d(0, \alpha) \not\equiv 0$, and $N(0, \alpha) \neq 0$ for each $\alpha \in \Gamma$. Thus $T_\alpha(s)$ has no fixed poles at $s = 0$, and not all entries of $T_\alpha(s)$ have a zero at $s = 0$. Finally we assume that

$$\frac{d\mathbf{y}}{d\alpha}(\alpha) = T_\alpha(0)\frac{d\mathbf{u}}{d\alpha}(\alpha), \quad \alpha \in \Gamma \tag{2}$$

It should be noted that that if $d(0, \alpha) \neq 0$ for each $\alpha \in \Gamma$, then (2) is a sufficient condition for $T_\alpha(s)$ with $\{[\mathbf{u}(\alpha), \mathbf{y}(\alpha)], \alpha \in \Gamma\}$ to be a linearization family for a nonlinear system. If the linearized state equation for the nonlinear system has no eigenvalues at zero for each $\alpha \in \Gamma$, then (2) also is necessary. [5]

Linearized Model Matching

The linearized model matching problem involves computing a nonlinear dynamic output feedback control law, with new input $w(t) \in R$, such that when the closed-loop nonlinear system is linearized about its constant operating point family, the resulting linearization family has a given parameterized transfer function. The given parameterized transfer function (model) for the linearized closed-loop system is written in a form similar to (1),

$$T_\alpha^m(s) = \frac{N^m(s, \alpha)}{d^m(s, \alpha)} \tag{3}$$

where the superscript m stands for model and $d^m(s, \alpha)$ is monic. We assume that for $\alpha \in \Gamma$ $d^m(0, \alpha) \neq 0$ and $N^m(0, \alpha) \neq 0$, so that $T_\alpha^m(s)$ has no pole at $s = 0$ and not all entries of $T_\alpha^m(s, \alpha)$ have a zero at $s = 0$. Also we assume that a family $\mathbf{w}(\alpha)$ of constant operating point values for the new input is given such that

$$\frac{d\mathbf{y}}{d\alpha}(\alpha) = T_\alpha^m(0)\frac{d\mathbf{w}}{d\alpha}(\alpha), \quad \alpha \in \Gamma \tag{4}$$

so that $T_\alpha^m(s)$ with $\{[\mathbf{w}(\alpha), \mathbf{y}(\alpha)], \alpha \in \Gamma\}$ is a linearization family.

The linearized model matching problem will be solved as follows. First we compute, if possible, a parameterized scalar linear precompensator $T_\alpha^c(s)$ such that

$$T_\alpha(s)T_\alpha^c(s) = T_\alpha^m(s) \tag{5}$$

This is illustrated in Figure 1 where the subscript δ indicates deviations from constant-operating-point values. Then we implement the precompensator $T_\alpha^c(s)$ using a parameterized linear dynamic output feedback configuration. Finally, a nonlinear, dynamic, output feedback law is computed whose family of linearizations is precisely the parameterized, linear, dynamic, output feedback law.

From (5) it is clear that $N(s, \alpha)$ and $N^m(s, \alpha)$ must satisfy

$$b(s, \alpha)N(s, \alpha) = a(s, \alpha)N^m(s, \alpha), \qquad \alpha \in \Gamma \tag{6}$$

for $a(s, \alpha)$ and $b(s, \alpha)$ which are scalar polynomials in s with coefficients that are real functions of α. If (6) is satisfied, the parameterized linear precompensator will have the form

$$T_\alpha^c(s) = \frac{b(s, \alpha)\, d(s, \alpha)}{a(s, \alpha)\, d^m(s, \alpha)}, \quad \alpha \in \Gamma \tag{7}$$

For $T^c_\alpha(s)$ to be proper we must have, for each $\alpha \in \Gamma$,

$$\delta d(s,\alpha) - \delta_i N(s,\alpha) \leq \delta d^m(s,\alpha) - \delta_i N^m(s,\alpha), \quad i = 1, \ldots, p \tag{8}$$

where δ denotes polynomial degree and δ_i denotes polynomial degree of the i^{th} row of a polynomial column vector. For example, if the plant and the model are such that $\delta d(s,\alpha) = \delta d^m(s,\alpha)$, then (8) implies that there should be no more finite zeros in the model $T^m_\alpha(s)$ than in the linearized plant $T_\alpha(s)$.

The precompensator $T^c_\alpha(s)$ is to be implemented by a parameterized, linear, dynamic, output feedback law. Although different implementations are available, a parameterized version of the implementation given in [2, pp 519-522] will be developed. This is presented in some detail because of additional hypothesis required due to parameterization by α.

Step 1: Assuming that $\delta a(s,\alpha)$ in (6) is constant for $\alpha \in \Gamma$, compute least degree polynomials $n_f(s,\alpha)$ and $d_f(s,\alpha)$ such that

$$\frac{n_f(s,\alpha)}{d_f(s,\alpha)} = \frac{T^c_\alpha(s)}{d(s,\alpha)} = \frac{b(s,\alpha)}{a(s,\alpha)d^m(s,\alpha)}, \quad \alpha \in \Gamma \tag{9}$$

Step 2: Letting $\delta d_f(s,\alpha) = f$ and $\delta d(s,\alpha) = n$, we set

$$\overline{d}_f(s,\alpha) = \nu(s,\alpha)d_f(s,\alpha), \qquad \overline{n}_f(s,\alpha) = \nu(s,\alpha)n_f(s,\alpha) \tag{10}$$

where $\nu(s,\alpha) = 1$ if $f \geq n$. If $f < n$, then let $\nu(s,\alpha)$ be an arbitrary, monic polynomial in s with coefficients that are real functions of α such that at each $\alpha \in \Gamma$, $\delta\nu(s,\alpha) = n - f$ and $\nu(s,\alpha)$ has only negative real part roots. Then we can write

$$\frac{\overline{n}_f(s,\alpha)}{\overline{d}_f(s,\alpha)} = \frac{T^c_\alpha(s)}{d(s,\alpha)}, \quad \alpha \in \Gamma$$

where $\delta\overline{d}_f(s,\alpha) \triangleq \overline{f} \geq n$.

Step 3: Assume that the observability index of a minimal parameterized linear realization of $T_\alpha(s)$, that is, the row index of $T_\alpha(s)$ is a constant ν for $\alpha \in \Gamma$. Let $d_c(s,\alpha)$ be a monic polynomial in s with coefficients that are real functions of α such that at each $\alpha \in \Gamma$, $\delta d_c(s,\alpha) \triangleq r \geq \nu - 1$, $\overline{n}_f(s,\alpha)/d_c(s,\alpha)$ is proper and $d_c(s,\alpha)$ has only negative real part roots. Set $w(s,\alpha) = 1$ if $\overline{f} \geq n + r$, and if $\overline{f} < n + r$, let $w(s,\alpha)$ be a monic polynomial in s with coefficients that are real functions of α such that at each $\alpha \in \Gamma$, $\delta w(s,\alpha) = n + r - \overline{f} \leq r$, and

$w(s, \alpha)$ has only negative real part roots.

Step 4: If $\delta[w(s, \alpha)\overline{d}_f(s, \alpha)] \le n + r$, set $q(s, \alpha) = 1$. Otherwise use polynomial division to write

$$\overline{d}_f(s, \alpha) = q_1(s, \alpha)d(s, \alpha) + r_1(s, \alpha), \qquad \delta r_1(s, \alpha) < \delta d(s, \alpha)$$

$$w(s, \alpha)q_1(s, \alpha) = d_c(s, \alpha)q(s, \alpha) + r_2(s, \alpha), \qquad \delta r_2(s, \alpha) < \delta d_c(s, \alpha) \tag{11}$$

In this division process it can happen that $q_1(s, \alpha)$ will have coefficients that are infinite at particular values of $\alpha \in \Gamma$. A similar singularity issue can arise for $q(s, \alpha)$, and also the possible occurrence of roots at $s = 0$ in $q(s, \alpha)$ complicates the subsequent calculation of nonlinear compensators. It is possible that these difficulties can be avoided by judicious choice of $d_c(s, \alpha)$. A solution in any case is to increase $r = \delta d_c(s, \alpha)$ to avoid the divisions in (11), though this does result in increased compensator dimension. (Note that choosing $d_c(s, \alpha)$, $v(s, \alpha)$ and $w(s, \alpha)$ to be independent of α will simplify matters.)

Step 5: Let

$$k(s, \alpha) = w(s, \alpha)\overline{n}_f(s, \alpha) \tag{12}$$

and determine polynomials $l(s, \alpha)$ and $M(s, \alpha)$ from

$$w(s, \alpha)\overline{d}_f(s, \alpha) - d_c(s, \alpha)q(s, \alpha)d(s, \alpha) = l(s, \alpha)d(s, \alpha) + M(s, \alpha)N(s, \alpha) \tag{13}$$

Step 6: Define the parameterized linear compensators in Figure 2 according to

$$C_\alpha^1(s) = \frac{k(s, \alpha)}{d_c(s, \alpha)} \tag{14}$$

$$C_\alpha^2(s) = \frac{1}{q(s, \alpha)} \tag{15}$$

$$C_\alpha^3(s) = \begin{bmatrix} \dfrac{l(s, \alpha)}{d_c(s, \alpha)} & \dfrac{M(s, \alpha)}{d_c(s, \alpha)} \end{bmatrix} \tag{16}$$

It is straightforward to verify that the parameterized transfer function for the system in Figure 2 is $T_\alpha^m(s)$. It should be noted that this parameterized linear system may not be minimal in the sense of linear systems. However, the above implementation of $T_\alpha^c(s)$ is such that the system will be internally stable so long as $T_\alpha^m(s)$ is stable.

The final task is to compute a nonlinear, dynamic, output feedback control law whose linearization family is precisely the parameterized, linear control law shown in Figure 2. Of course, the closed-loop, parameterized linear system in Figure 2 described by $T_\alpha^m(s)$ together with $\{[\mathbf{w}(\alpha), \mathbf{y}(\alpha)], \alpha \in \Gamma\}$ is a linearization family, and $T_\alpha(s)$ with $\{[\mathbf{u}(\alpha), \mathbf{y}(\alpha)], \alpha \in \Gamma\}$ is a linearization family. Now we define constant operating point values according to

$$\mathbf{r}(\alpha) = \int_{\alpha_0}^{\alpha} C_\sigma^1(0)\frac{d\mathbf{w}}{d\sigma}(\sigma)d\sigma, \quad \mathbf{e}(\alpha) = \int_{\alpha_0}^{\alpha} \frac{1}{C_\sigma^2(0)}\frac{d\mathbf{u}}{d\sigma}(\sigma)d\sigma$$

$$\mathbf{v}(\alpha) = \int_{\alpha_0}^{\alpha} C_\alpha^3(0)\begin{bmatrix} \frac{d\mathbf{u}}{d\sigma}(\sigma) \\ \frac{d\mathbf{y}}{d\sigma}(\sigma) \end{bmatrix} d\sigma \qquad (17)$$

so that each parameterized linear compensator with its associated constant operating point family forms a linearization family. Using (2) and (4) it is not difficult to verify $\mathbf{e}(\alpha) = \mathbf{r}(\alpha) - \mathbf{v}(\alpha)$ explicitly, that is, these definitions are consistent. The construction of nonlinear compensators from these compensator linearization families is discussed in [5], and will be illustrated in an example.

In order to summarize the foregoing development, we state the following theorem.

Theorem: Suppose we are given a model (3) and a nonlinear system with linearization family (1) such that, for $\alpha \in \Gamma$, where Γ is a sufficiently small neighborhood of α_0, $d(0, \alpha) \not\equiv 0$, $N(0, \alpha) \neq 0$, $d^m(0, \alpha) \neq 0$, $N^m(0, \alpha) \neq 0$, and (2), (4) hold. Then there is a dynamic output feedback control law that solves the linearized model matching problem if for $\alpha \in \Gamma$

i) $T_\alpha(s)$ is irreducible;

ii) the row index, ν, of $T_\alpha(s)$ is constant;

iii) $N(s, \alpha)b(s, \alpha) = a(s, \alpha)N^m(s, \alpha)$ for scalar polynomials $a(s, \alpha)$ and $b(s, \alpha)$, where $\delta a(s, \alpha)$ is constant;

iv) $\delta d(s, \alpha) - \delta_i N(s, \alpha) \leq \delta d^m(s, \alpha) - \delta_i N^m(s, \alpha)$, $i = 1, \ldots, p$.

Example: The dynamics of an inverted pendulum on a cart are described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ \dfrac{g\sin(x_1) - amlx_2^2\sin(x_1)\cos(x_1) - a\cos(x_1)u}{4l/3 - aml\cos^2(x_1)} \end{bmatrix}$$

$$y = x_1$$

where x_1 is the angle of the pendulum from the vertical, u is the force applied to the cart, $a = 1/(m + M)$, m is the pendulum mass, M is the cart mass, $2l$ is the pendulum length and g is the acceleration due to gravity. The constant operating point family is easily parameterized by the first component of the state:

$$\mathrm{u}(\alpha) = \frac{g}{a}\tan(\alpha), \quad \mathrm{x}(\alpha) = \begin{bmatrix} \alpha \\ 0 \end{bmatrix}, \quad \mathrm{y}(\alpha) = \alpha; \quad \alpha \in \Gamma \triangleq (-\frac{\pi}{2}, \frac{\pi}{2}), \quad \alpha_0 = 0$$

The transfer function of the linearized system is

$$T_\alpha(s) = \frac{c(\alpha)}{s^2 - b(\alpha)}$$

where

$$b(\alpha) = \frac{g}{[4l/3 - aml\cos^2(\alpha)]\cos(\alpha)}, \quad c(\alpha) = \frac{-a\cos(\alpha)}{4l/3 - aml\cos^2(\alpha)}$$

Note that $T_\alpha(s)$ is irreducible and has neither pole nor zero at $s = 0$ for any $\alpha \in \Gamma$ since $4l/3 > aml$, and also that (2) is satisfied. Let the model be specified by

$$T_\alpha^m(s) = \frac{1}{(s+1)^2}, \quad \mathrm{w}(\alpha) = \alpha$$

Obviously (4) is satisfied, and the precompensator is given by

$$T_\alpha^c(s) = \frac{s^2 - b(\alpha)}{c(\alpha)(s+1)^2}$$

To implement $T_\alpha^c(s)$ by the feedback configuration in Figure 2, we apply the algorithm as follows.

Step 1: Let $n_f(s, \alpha) = 1$, $d_f(s, \alpha) = c(\alpha)(s+1)^2$.

Step 2: Since $\delta d_f(s, \alpha) = f = 2$, $\delta d(s, \alpha) = n = 2$, let $v(s, \alpha) = 1$, and set $\overline{d_f}(s, \alpha) = d_f(s, \alpha)$, $\overline{n_f}(s, \alpha) = n_f(s, \alpha)$, with $\overline{f} = f = 2$.

Step 3: Since $\nu = n = 2$, let $r = 1$, and $d_c(s, \alpha) = s + 1$. Then $\overline{f} < n + r$, and we let $w(s, \alpha) = s + 1$.

Step 4: Since $\delta[w(s, \alpha)\overline{d_f}(s, \alpha)] = n + r$, let $q(s, \alpha) = 1$.

Step 5: Let $k(s, \alpha) = w(s, \alpha)\overline{n}_f(s, \alpha) = s + 1$, and solve (12) to obtain

$$l(s, \alpha) = [c(\alpha) - 1]s + 3c(\alpha) - 1$$

$$M(s, \alpha) = [b(\alpha) + 3]s + 3b(\alpha) + 1$$

Step 6: The parameterized linear compensators in Figure 2 are easily computed to be

$$C_\alpha^1(s) = C_\alpha^2(s) = 1$$

$$C_\alpha^3(s) = \left[\frac{[c(\alpha) - 1]s + 3c(\alpha) - 1}{s + 1} \quad \frac{[b(\alpha) + 3]s + 3b(\alpha) + 1}{s + 1} \right]$$

From (17), $\mathbf{r}(\alpha) = \mathbf{w}(\alpha)$, $\mathbf{e}(\alpha) = \mathbf{u}(\alpha)$, and $\mathbf{v}(\alpha) = \alpha - (g/a)\tan(\alpha)$. Thus the nonlinear compensators associated with $C_\alpha^1(s)$ and $C_\alpha^2(s)$ can be taken as unity gains. A parameterized linear realization for the compensator $C_\alpha^3(s)$ is

$$\dot{\tau}_\delta = -\tau_\delta + \begin{bmatrix} 2c(\alpha) & 2(b(\alpha) - 1) \end{bmatrix} \begin{bmatrix} u_\delta \\ y_\delta \end{bmatrix}, \qquad \tau(\alpha) = -2\alpha$$

$$v_\delta = \tau_\delta + \begin{bmatrix} c(\alpha) - 1 & b(\alpha) + 3 \end{bmatrix} \begin{bmatrix} u_\delta \\ y_\delta \end{bmatrix}, \qquad \mathbf{v}(\alpha) = \alpha - \frac{g}{a}\tan(\alpha)$$

Finally, it is easy to check that a corresponding nonlinear compensator is described by [5]

$$\dot{\tau} = -\tau - 2y + \frac{2g\sin(y) - 2au\cos(y)}{4l/3 - aml\cos^2(y)}$$

$$v = \tau - u + 3y + \frac{g\sin(y) - au\cos(y)}{4l/3 - aml\cos^2(y)}$$

Remark: Linearized model matching can be generalized to the case where $T_\alpha(s)$ and $T_\alpha^m(s)$ have poles at $s = 0$ that are invariant with respect to $\alpha \in \Gamma$. The main changes to be made involve a more complicated characterization of the associated constant operating points for the linearization families. For example, suppose $T_\alpha(s)$ is such that that

$$d(s, \alpha) = (s^{n-1} + d_1(\alpha)s^{n-2} + \cdots + d_{n-1}(\alpha))s$$

$$\triangleq \hat{d}(s, \alpha)s, \quad d_i(\alpha) \in R, \quad d_{n-1}(\alpha) \neq 0 \text{ for } \alpha \in \Gamma \tag{18}$$

Then we must have $\mathbf{u}(\alpha) = 0$ for $\alpha \in \Gamma$. To characterize the associated constant operating point output, write $T_\alpha(s)$ as the concatenation of

$$\bar{T}_\alpha(s) = \frac{N(s,\alpha)}{\hat{d}(s,\alpha)(s+1)} \tag{19}$$

with output y and input $\bar{u}$, and

$$\hat{T}_\alpha(s) = \frac{s+1}{s} \tag{20}$$

with input u and output $\bar{u}$. It is easy to see that $T_\alpha(s)$ with $[\mathbf{u}(\alpha), \mathbf{y}(\alpha)]$ is a linearization family if and only if $\bar{\mathbf{u}}(\alpha)$ can be found such that $\bar{T}_\alpha(s)$ with $[\bar{\mathbf{u}}(\alpha), \mathbf{y}(\alpha)]$, and $\hat{T}_\alpha(s)$ with $[\mathbf{u}(\alpha), \bar{\mathbf{u}}(\alpha)]$ both are linearization families. Since $\hat{T}_\alpha(s)$ with $\mathbf{u}(\alpha) = 0$ and any $\bar{\mathbf{u}}(\alpha)$ is a linearization family, $T_\alpha(s)$ with $\{[\mathbf{u}(\alpha) = 0, \mathbf{y}(\alpha)], \alpha \in \Gamma\}$ is a linearization family if $\bar{\mathbf{u}}(\alpha)$ can be found such that

$$\frac{d\mathbf{y}}{d\alpha}(\alpha) = \bar{T}_\alpha(0)\frac{d\bar{\mathbf{u}}}{d\alpha}(\alpha), \quad \text{for } \alpha \in \Gamma \tag{21}$$

Conclusions

The linearized model matching problem is a natural extension of eigenvalue placement problems formulated in terms of extended linearization [1] or pseudo-compensation [4] in that numerator gains and zero locations of the linearized closed-loop system are explicitly addressed. This additional capability can be useful when acceptable performance cannot be achieved by assignment of linearized-system eigenvalues alone.

If a parameterized compensator can be found such that the model can be taken to be independent of α, as in the Example, then the linearized model matching problem leads to what might be called *input-output pseudolinearization*. However this bears only loose resemblance to the original notion of pseudo-linearization in that variable changes are not involved, full state feedback is not used, and the dimension and eigenvalue locations of the closed-loop linearization family are not specified a priori.

One disadvantage of the frequency-domain approach used here is that extension of the results to the multi-input case, though clear in broad outline, is difficult in detail. For example, suppose the parameterized linear compensators are specified in terms of parameterized transfer function matrices. In the course of constructing the nonlinear compensators, computation of parameterized linear realizations of the transfer function descriptions can lead to singularities with respect to the parameter vector α that are not easily shown to be removable.

Also, the so-called integrability conditions for the existence of a nonlinear control law corresponding to a parameterized linear control law are nontrivial in the multi-input case.

References

[1] W.T. Baumann, W.J. Rugh, "Feedback Control of Nonlinear Systems by Extended Linearization," *IEEE Transaction of Automatic Control*, Vol. AC-31, No. 1, pp. 40-46, 1986.

[2] C.T. Chen, *Linear System Theory and Design*, Holt, Rinehart and Winston, New York, 1984.

[3] C. Reboulet, C. Champetier, "A New Method for Linearizing Nonlinear Systems: the Pseudo-linearization," *International Journal of Control*, Vol. 40, No. 4, pp. 631-638, 1984.

[4] C. Reboulet, P. Mouyon, C. Champetier, "About the Local Linearization of Nonlinear Systems," *Algebraic and Geometric Methods in Nonlinear Control Theory*, M. Fliess, M. Hazewindel, eds, D. Reidel, Dordrecht, Holland, 1986.

[5] J. Wang, W.J. Rugh, "Parameterized Linear Systems and Linearization Families for Nonlinear Systems," *IEEE Transactions on Circuits and Systems*, Vol. CAS-34, No. 6, pp. 650-657, 1987.

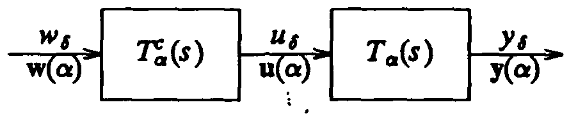


Figure 1. Linearized plant with precompensator.

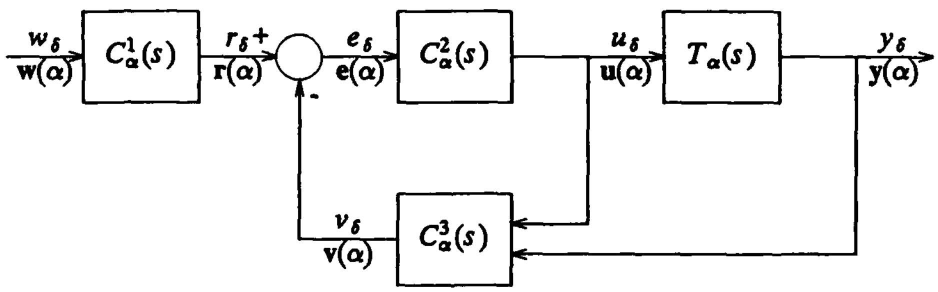


Figure 2. Feedback implementation of precompensator.

END
DATE
FILMED
7-88
DTIC